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Evidence for the Cosmological Evolution of Active Galaxies [and Discussion]

J. V. Wall and D. Meier

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Evidence for the cosmological evolution of active galaxies

By J. V. Wall
Mullard Radio Astronomy Observatory, Cavendish Laboratory, Cambridge CB3 0HE, U.K.

Investigations of the radial distribution of quasars and radio galaxies attest to a period of relatively violent activity at earlier epochs of the Universe. The evidence is from applications of the luminosity–volume (V/V_{max}) test, and from interpretations of source counts at both radio ('log N–log S' curve) and optical ('log $N-m$' curve) wavelengths. Here these techniques are discussed in the light of modern data. In addition to confirming the existence of powerful cosmic evolution, these data allow some detail of the evolution to be delineated, and indicate marked differences among the evolutionary behaviours of different populations of active galaxies. Some physical considerations are briefly described.

In this contribution I take the term 'active galaxies' to encompass both quasi-stellar objects (quasars) and radio galaxies, since it is widely (but not universally) accepted that quasars are superactive nuclei of galaxies. It is also widely accepted that radio galaxies and quasars are powered by a nuclear engine, which provides efficient energy conversion in a process (somehow) involving a massive rotator to produce the observed gyroscopic and collimating effects. Massive black holes accreting gas or whole stars have been advertised as the 'best buy' in the current market (Rees 1977). The cosmic history of such nuclear activity is therefore mapped by studies of the radial distribution of active galaxies, and it is the purpose of this paper to summarize results of these studies.

For the most part, the results rely on the cosmological interpretation of red shifts, and I indicate which conclusions stand if the assumption is abandoned. My summary lacks some of the analytical apparatus necessary for the studies, and this may be found together with much more of relevance in reviews by Longair (1971, 1978) and Scheuer (1975). At the outset, I emphasize that there are other ways of exploring the behaviour of active galaxies with epoch which complement spatial-distribution studies: the angular size – flux density (θ–S) and the angular size – red shift (θ–z) relations (see, for example, Miley 1968; Wardle & Potash 1977; Kapahi 1975; Ekers & Miley 1977), and the Hubble diagram for quasars (see, for example, Bahcall & Hills 1973; Turner 1979). These are beyond the scope of this review.

1. The tools and basic results: evolution

There are two basic tools for examining the radial distribution of active galaxies: the luminosity–volume (V/V_{max}) test (Schmidt 1968; Rowan-Robinson 1968) and the source–count ($N(S)$ or $N(m)$) test (Mills 1952; Bolton *et al.* 1954; Ryle & Scheuer 1955; Ryle & Clarke 1961). The former is less controversial, so I begin with it.

1.1. *The luminosity–volume test*

Consider a set of N objects constituting a complete sample over a region of sky in the sense that for each object, apparent intensity S_i (radio or optical wavelengths) exceeds some intensity limit S_0. Suppose that redshifts z_i and continuum spectra (designated by spectral index α_i where $S_i \propto \nu^{-\alpha_i}$) are known. The red shift enables $V(z_i)$, the comoving volume out to red shift z_i, to be calculated from

$$\mathrm{d}V(z) = (\sin Ar/A)^2 \,\mathrm{d}r, \tag{1}$$

where $\sin Ar/A$ contains all information on the assumed geometry (Scheuer 1975). Suppose then that the object is 'pushed' radially outward; the observed (S_i, z_i, α_i) yield an intrinsic luminosity P_i at the frequency of observation via

$$S_\nu = P_\nu/(\sin Ar/A)^2 \,(1+z)^{1+\alpha} \tag{2}$$

and we push the object until (2) says that at some $z = z_i(\max)$, the observed intensity has dropped to S_0, the survey limit. From $z_i(\max)$, calculate $V_i(\max)$, the maximum volume inside which the object can be observed. If the objects are uniformly distributed, then no object should have a predilection for any region of its own $V_i(\max)$. Hence the quantity $(V/V(\max))_i$ should be uniformly distributed between 0 and 1, and $\langle V/V_{\max}\rangle$ should be $0.5 \pm (12N)^{-\frac{1}{2}}$. The test is easily generalized when more than one survey limit is involved, as in detecting a sample of radio sources above a (radio) survey limit and subsequently identifying optical counterparts above some (optical) survey limit. The appropriate $V_i(\max)$ corresponds to the *first* limit encountered in the pushing process.

The test is simple in concept; the difficulties arise in application. It is laborious to use in that *each* object must have a measured red shift, accurate intensity measurements at each observational frequency relevant to sample definition, and continuum measurements in each of these wavebands to get the spectral indices which we require to find $z(\max)$ from (2). Because the surface density – apparent intensity relations ($N(S)$ in radio parlance; $N(m)$ in optical) are very steep, it is particularly important to avoid systematic errors in the intensity measurements. Finally the objects must be very luminous – and if two or more limits are involved, they must be very luminous in each waveband – so that the survey limit(s) admitting enough objects to a sample correspond to red shifts of some cosmological significance. This requirement has limited the use of the test to quasars for the most part. Indeed, in addition to being less luminous in the optical waveband, radio galaxies suffer two other observational disadvantages for $V/V_{\max}$ tests; both the magnitudes and the red shifts are harder to measure.

Most of the samples on which $V/V_{\max}$ analyses have been performed are collected in table 1. Without exception, the $\langle V/V_{\max}\rangle$ exceed 0.5. In not all cases do they exceed 0.5 by more than 1σ, and for some types of object the excess is significantly less than for other samples. This 'differential evolution' is discussed in §2. But for samples composed of quasars of the 'steep-spectrum' type found in low-frequency surveys (3CR, 4C), the results are unequivocal and have withstood considerable cross-examination: the large $V/V_{\max}$ values indicate a highly non-uniform radial distribution, a preference of quasars for the more distant regions of their accessible volumes. Just what the density enhancement is as a function of red shift or epoch depends on the density- modifying 'evolution function' adopted, but typically $\langle V/V_{\max}\rangle = 0.70$ implies an increase over present-day numbers at $1 < z < 3$ of *ca.* 1000, as Longair (1978) shows with a simple analysis.

In addition to demonstrating the existence of evolution, much can be learned from $V/V_{\max}$ data on the form of the luminosity function itself as well as how it has evolved; this discussion is postponed to §2.1.

TABLE 1. RESULTS FROM THE LUMINOSITY–VOLUME (V/V_{max}) TEST

reference	sample†	number	$\langle V/V_{max}\rangle$	limits
Schmidt (1968)	quasars from 3CR survey; predominantly 'steep-spectrum' type	33	0.70 ± 0.05	$S_{178} = 9$ Jy;§ $V = 18.4$
Lynds & Wills (1972)	quasars from 4C survey; predominantly 'steep-spectrum' type	31	0.67 ± 0.05	$S_{178} = 2$ Jy; $V = 19.4$
Fanti *et al.* (1973)‖	3CR } as above;	33	0.70 ± 0.05	$S_{178} = 9$ Jy; $m_v = 18.4$
	4C } as above;	30	0.67 ± 0.05	$S_{178} = 2$ Jy; $m_v = 19.5$
	optically selected quasars	23	0.63 ± 0.06	$m_i = 17.65$
Schmidt (1976)	quasars from N.R.A.O. 5 GHz survey:			
	$\alpha < 0.05$	17	0.52 ± 0.06	not specified
	$0.05 < \alpha < 0.45$	17	0.60 ± 0.07	not specified
	$\alpha > 0.45$	17	0.71 ± 0.05	not specified
Masson & Wall (1977)‡	quasars from PKS 2.7 GHz survey:			
	$\alpha < 0.0$	16	0.50 ± 0.07	$S_{2700} = 0.35$ Jy; $B = 19.5$
	$0.0 < \alpha < 0.5$	26	0.57 ± 0.06	$S_{2700} = 0.35$ Jy; $B = 19.5$
	$\alpha > 0.5$	15	0.68 ± 0.07	$S_{2700} = 0.35$ Jy; $B = 19.5$
Schmidt (1977)‖	optically selected quasars:			
	Sandage & Luyten	34	0.66 ± 0.05	not specified
	Braccesi & Formiggini	20	0.70 ± 0.06	not specified
Wills & Lynds (1978)	3CR quasars, predominantly 'steep-spectrum'	34	0.69 ± 0.04	$S_{178} = 9$ Jy; $m_v = 19.5$
	4C quasars, predominantly 'steep-spectrum'	76	0.70 ± 0.02	$S_{178} = 2$ Jy; $m_v = 19.5$
	quasars from PKS 2.7 GHz survey:			
	$\alpha < 0.0$	18	0.62 ± 0.06	$S_{2700} = 0.35$ Jy; $m_v = 19.5$
	$0 < \alpha < 0.5$	27	0.60 ± 0.05	$S_{2700} = 0.35$ Jy; $m_v = 19.5$
	$\alpha > 0.5$	15	0.76 ± 0.05	$S_{2700} = 0.35$ Jy; $m_v = 19.5$
Laing *et al.* (1978)	3CR quasars, predominantly 'steep-spectrum'	23	0.71 ± 0.06	$S_{178} = 10$ Jy; $V = 23$
	3CR radio galaxies:			
	$P_{178} > 10^{26}$ W Hz^{-1} sr^{-1}	23	0.61 ± 0.06	$S_{178} = 10$ Jy; $V = 23$
	$P_{178} < 10^{26}$ W Hz^{-1} sr^{-1}	10	0.52 ± 0.08	$S_{178} = 10$ Jy; $V = 23$
Savage (1978)	optically selected quasars with u.v. excess	148	0.60 ± 0.02	$b = 20$

† Radio spectral index α is defined in the sense $S \propto \nu^{-\alpha}$.
‡ $\langle V/V_{max}\rangle$ values of Masson & Wall corrected for computational errors (Wills & Lynds 1978).
§ 1 Jy (jansky) $= 10^{-26}$ W m^{-2} Hz^{-1}.
‖ Optically selected quasars of Fanti *et al.* (1973) and Schmidt (1977) are from Sandage & Luyten (1969) and Braccesi & Formiggini (1969).

1.2. *The source–count test*

In contrast to V/V_{max} data, source–count data are easy to obtain: they are the fundamental data from radio or optical surveys, the surface density on the sky as a function of apparent intensity. The price paid for this simplicity is an interpretation that is less obvious, and which in the earlier days of radio counts resulted in a notorious controversy. Distances to the sources constituting the counts were not known *a priori*, and indeed the nature of the objects (extragalactic versus galactic) was queried; add to this the horrific effects of confusion which pioneering source–counters failed to appreciate and the emergence at the appropriate time of the steady-state model for the Universe, and controversy was inevitable.

New observations, particularly optical identifications, and new interpretations have revealed the wealth of information that the radio counts do contain. Instrumental effects are now well understood, and radio-counts that are relatively free of these are available at several frequencies. A compilation is shown in figure 1, a current version of an earlier compilation (Wall 1978, fig. 6). The changes have been in the sense of decreasing the differences between the counts at different frequencies; but a significant change in form persists, reflecting the different radio-source populations which are preferentially selected in surveys at different frequencies. All

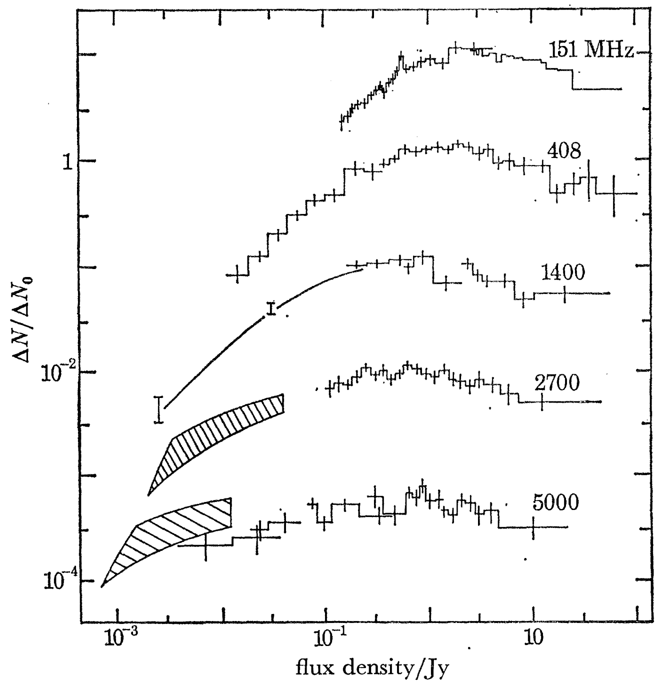

FIGURE 1. Counts of radio sources at five frequencies in 'relative differential' form: ΔN is the number of sources per steradian with flux densities between S and $S+\Delta S$, and ΔN_0 is the number expected on the basis of a static Euclidean Universe in which sources are uniformly distributed, i.e. $N_0 \propto S^{-1.5}$ and $\Delta N_0 = k\,[S^{-1.5} - (S+\Delta S)^{1.5}]$, where k is an arbitrary constant. Error bars describe $(\Delta N)^{-\frac{1}{2}}$ statistical uncertainties. This diagram is an updated version of a previous compilation (fig. 6 of Wall 1978), where references may be found to the surveys from which the counts were compiled. New data here consist of revised counts at faint levels for 151 MHz (Waggett 1978) and for 5000 MHz (Pauliny-Toth *et al.* 1978; Willis & Miley 1979). These latter references suggest that the statistical definition of the faint end of the 5000 MHz count (Wall 1978) is an overestimate by *ca.* 2σ.

the counts, however, are of the following form in the 'relative differential' presentation of figure 1: with decreasing flux density, there is an initial rise, a maximum, and a tail-off toward the faintest flux densities to which surveys have been carried. Translating this back into integral form, i.e. the 'log N–log S' curve: the initial slope is steeper than the $-\frac{3}{2}$ power law which the static, uniform, Euclidean Universe must show, and the slope changes gradually to a law flatter than the $-\frac{3}{2}$ law – as indeed it must if we are to escape the radio equivalent of Olbers's Paradox.

The counts predicted for universes of conventional (Friedmann) geometries in which radio sources are uniformly distributed look nothing like the curves of figure 1. For *any* luminosity, a count computed on these assumptions is asymptotic to a $-\frac{3}{2}$ power law (or to a horizontal line in the relative differential form of figure 1) at $S = \infty$, and drops monotonically with decreasing flux density (see Scheuer (1975) and fig. 16 of Longair 1978). The point is emphasized in figure 2, a count obtained at 408 MHz on which is superposed a count computed for a source-conserving Universe, one in which radio sources obey a uniform radial distribution. This curve is in fact the sum of counts computed for different luminosities, added together in proportions yielding the correct distribution of luminosities among the bright sources (Wall *et al.* 1977). The computed curve is based on cosmological red shifts, but appeals to non-cosmological red shifts are of little help: if we postulate luminosities much lower than red

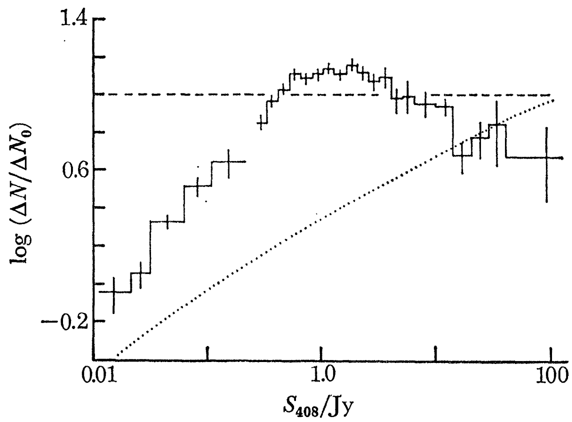

FIGURE 2. The 408 MHz source count in 'relative differential' form, the broken horizontal line representing the count anticipated for a static, Euclidean Universe with uniform radial distribution of sources (cf. figure 1). The dotted line is a count computed for a uniform distribution of sources in a realistic geometry, a Friedmann universe with density parameter $\Omega = 1$. This count is the result of combining counts for individual radio luminosities in proportions to yield the observed identification/red shift data for the brighter ($S_{408} \geqslant 10$ Jy) sources (Wall *et al.* 1977). The computed count is normalized to give the (integral) surface density observed at $S_{408} = 10$ Jy.

shift distances suggest, then we must add (in appropriate proportions) families of curves lying between the computed curve and the horizontal line of figure 2. Obviously it is impossible to reproduce the observed count in this manner, and it is important to note that it is the *hump* of the observed count that cannot be mirrored, not just the initial steep portion over which the early battles were waged. Other ways have been proposed to make the counts conform to uniform radial distributions: exotic cosmologies, a local hole, or attributing the unidentified sources to a nearby 'mystery population'. These options become hopelessly contrived in the light of (i) analyses showing that radio sources at all accessible flux levels are uniformly, randomly and independently distributed on the celestial sphere (see, for example, Webster 1977) and (ii) modern optical data resulting in near-complete identifications for some samples of bright radio sources.

The counts tell us then that something drastic has happened to the spatial distribution of radio sources with epoch. To find out what, numerical analyses are carried out along the following line. Consider a single type of source, spectral index α, and the so-called 'evolution plane', the radio luminosity–red shift (P–z) surface. Each element (P_i, z_i) of this plane contributes to the source count a total of

$$\Delta n(P_i, z_i) = \tfrac{1}{4}\pi^{-1}\rho(P_i, z_i)\, \Delta V_i\, \Delta P_i \tag{3}$$

sources per steradian of flux density S_i given by (2) for (P_i, z_i), where ΔV_i is the volume in red shift shell Δz_i (equation 1), and ρ is the luminosity function. This latter may be written with complete generality as

$$\rho(P, z) = F(P, z)\, \rho_0(P), \tag{4}$$

with ρ_0 as the 'local' luminosity function, i.e. $\rho_0 = \rho(P, z = 0)$, and $F(P, z)$ the 'evolution function', the density enhancement in each (P_i, z_i) element. Calculation of the count is simple: it consists of summing the flux-density contributions Δn over all P_i in the luminosity function, and out to z_i large enough so that the highest P_i produce flux densities at the very faintest levels of interest. In addition to $F(P, z)$, we must assume ρ_0 and the geometry. The geometry does not matter particularly, since evolutionary effects overwhelm it anyway. The local luminosity function must be chosen to agree with identification/red shift data; Wall *et al.* (1977) describe a count-computing technique which avoids assuming ρ_0 and which forces agreement with the identification data of the bright sources.

The pioneering investigation of source counts in this numerical fashion is Longair's (1966) analysis of 178 MHz (3CR and 4C) data, although the basic results were obtained in the earlier analytical investigations of Oort (1961) and Davidson (1962). The essential result, agreed in these and later investigations (e.g. Rowan-Robinson 1970*a*; Ringenberg & McVittie 1970), is that very powerful evolution is needed to shape the hump of the differential count, the comoving densities at epochs corresponding to $1 < z < 4$ exceeding present-day densities by factors not less than 1000. The strength of this evolution is quantitatively similar to that derived from V/V_{max} analyses of quasars selected in low-frequency (3CR and4C) radio surveys. This is no surprise; Longair & Scheuer (1970) showed analytically why V/V_{max} and source-counts contain the same message, and in any case the objects constituting the V/V_{max} samples are also present at the bright end of the source counts.

The source-count test has been applied to surface densities of optically selected quasars (the number-magnitude or $N(m)$ relation). Since $m_2 - m_1 = 2.5 \lg (S_1/S_2)$, the $-\frac{3}{2}$ integral law, $\lg N\,(>S) = -\frac{3}{2} \lg S + K_1$, translates to $\lg N\,(>m) = 0.6\, m + K_2$ optical in terms. Braccesi & Formiggini (1969) and Sandage & Luyten (1969) estimated values of the $N\,(>m)$ slope in the range $17.5 \lesssim m \lesssim 19$ as 0.72 and 0.75 respectively. Setti & Woltjer (1973) suggested that magnitude errors and the small range of magnitude made these results uncertain to the extent that a slope of 0.6 was permitted. But again it must be stressed that there is nothing magic about such a slope – when a reasonable geometry is applied, the initial slope expected for uniform distribution is *ca.* 0.4, very much less than 0.6. Recently Green & Schmidt (1978) obtained a slope of 0.93 (corresponding to a log N–log S slope of -2.3) over the greatly increased range of $15.7 < B < 18.5$. It therefore seems essential to postulate a non-uniformity in the spatial distribution of optically selected quasars which is as strong or stronger than that of the radio quasars in low-frequency surveys: the numbers at earlier epochs were very much greater than at present. Green & Schmidt's paper, entitled 'Evidence for the nonuniform radial distribution of quasars, regardless of the nature of their redshifts', is in fact a concise exposition of the source-count test.

2. More results: luminosity functions, evolution functions, differential evolution and the Dark Ages

The basic tools for exploring radial distribution of active galaxies have been adapted to yield far more than qualitative claims of evolution.

2.1. *Luminosity functions*

Particularly direct information is yielded by V/V_{max} data: each object i contributes $1/V_i(\max)$ in space density to its particular luminosity bin (radio or optical) at its red shift z_i. Thus the so-called 'bivariate luminosity function' $\rho(P_{rad}, P_{opt})$ can be constructed, in principle at any epoch, in practice (because of limited numbers) at one epoch by correcting with an adopted evolution function (see, for example, Lynds & Wills 1972; Fanti *et al.* 1973). Summing the rows (or the columns) of the bivariate distribution yields $\rho(P_{opt})$ or $\rho(P_{rad})$, and the forms of these have been investigated by Wills & Lynds (1978). For the steep-spectrum radio quasars of the low-frequency surveys, $\lg \rho(P_{rad}, z = 1) \propto -1.5 \lg P_{rad}$, while the optical luminosity function is approximately flat. A different behaviour is evident for the radio quasars of compact structure found in higher-frequency surveys (Masson & Wall 1977; Wills & Lynds 1978); the radio luminosity function flattens ($\lg \rho(P_{rad}, z = 1) \propto -0.8 \lg P_{rad}$) while the optical luminosity function steepens to $\lg \rho(P_{opt}, z = 1) \propto -0.5 \lg P_{opt}$. The differences between spatial distributions of the 'extended' and 'compact' radio quasars are discussed below.

The overall form of the bivariate luminosity function is of great interest because of its astrophysical implications. Schmidt (1970) showed that the simplest formulation, $\rho(P_{opt}, P_{rad}, z) = \rho(P_{opt})\, \rho(P_{rad})\, F(z)$, is unacceptable because the red shift distribution of radio-quiet quasars is approximately the same as that of the radio quasars while the apparent magnitude distributions differ. In its stead he proposed the factorization $\rho = \rho(P_{opt})\, \psi(P_{rad}/P_{opt})\, F(z)$, where ψ is a universal 'colour function', $\psi \neq \psi(z)$. This formulation results in a radio luminosity function which depends on optical luminosity (the reverse does not hold): the mean radio luminosity increases with increasing optical luminosity to ensure that the red shift distribution depends on the *optical limit* but not on the *radio limit*. The implication that the radio power depends on the optical is startling at first, since the radio emission for the extended quasars in 3CR and 4C surveys is generally displaced from the nucleus in the standard double-lobe pattern. In the context of the nuclear engine it is not so surprising: perhaps increased rate of energy supply by the engine to the lobes is reflected in increased optical activity. But it should be emphasized that the formulation describes the spatial distribution of steep-spectrum quasars; there remains uncertainty about whether or not $\psi(R)$ *is* independent of red shift, whether or not the formulation encompasses the flat-spectrum quasars and the quasars of very low ratios of radio:optical luminosity (see, for example, Fanti *et al.* 1973, 1975).

2.2. *Evolution functions*

In the foregoing formulation, the evolution function F is a function of z alone, and this is referred to as 'density evolution'. It has been used in most V/V_{max} analyses in preference to 'luminosity evolution'. Indeed Schmidt (1972*a*) showed that *pure* luminosity evolution, in which numbers of quasars per volume shell have remained constant but their mean luminosity has decreased with cosmic time, is incompatible with the small change observed in the red shift distribution with magnitude. (The conclusion has been disputed by Mathez (1976).)

The simplification of $F = F(z)$ is not possible in source-count analysis; because of the much larger luminosity range it is essential to use $F = F(P_{rad}, z)$, but this has considerable rewards, as I describe in §2.3. In the meantime we shall accept that F for the high-power radio sources may be written as $F(z)$, and ask what forms of $F(z)$ have been deemed satisfactory from both source-count and V/V_{max} analyses. Three *ad hoc* functions appear in the literature:

$$F(z) = (1+z)^n,\ 5 < n < 7 \quad (\text{'power law' evolution})$$

$$= \exp[(M/t_0)(t_0-t)],\ 9 < M < 12 \quad (\text{'exponential' evolution})$$

$$= V^a,\ 0.6 < a < 1.5 \quad (\text{'volume' evolution})$$

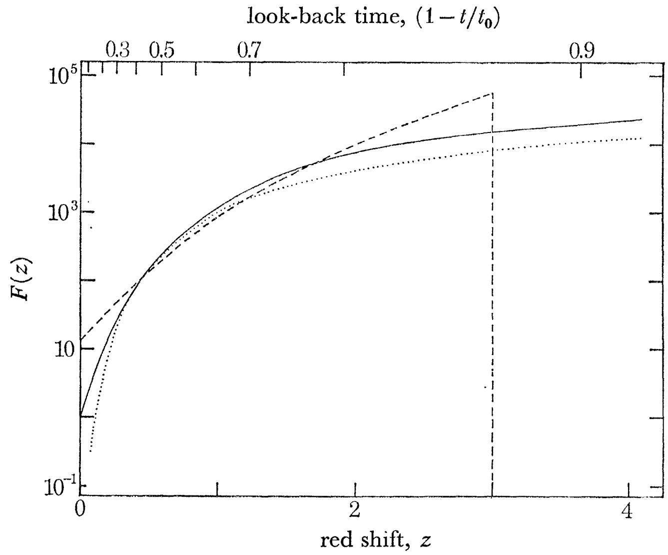

FIGURE 3. The behaviour of *ad hoc* evolution functions in a Friedmann Universe with $\Omega = 1$; the curves describe density enhancement of the 'steep-spectrum' radio quasars and powerful radio galaxies as a function of red shift/epoch. (Note that t is the age of the Universe at z, and t_0 is the present age.) Solid curve, $F(z) = \exp[11.0(1-t/t_0)]$; broken curve, $F(z) = (1+z)^{6.0}$, cutoff $z = 3.0$; dotted curve, $F(z) = V^{1.33}$. All curves have been normalized to give the same density enhancement at $z = 0.5$.

The first of these was proposed by Longair (1966) from source-count analysis. Its appeal lies in the fact that it is the scale factor of the Universe to some (negative) power, and as such *might* have physical relevance via density or temperature of intergalactic matter (i.g.m.), or via energy density of the microwave background. It was quickly adopted in V/V_{max} analyses (Schmidt 1968) and both types of analysis agree that such a law must be truncated or at least flattened for $z \geqslant 2.5$. This feature also had its attractions because at the time no larger red shifts had been observed. However, it was soon recognized that *ad hoc* functions were available that did not require modification at any red shift, and one of these is the 'exponential' law (Rowan-Robinson 1970*a*; Doroshkevich *et al.* 1970; Schmidt 1970) in which t_0 is the age of the Universe, and (t_0-t) is the look-back time to epoch $t(z)$. In this formulation, $t_0/M \approx 0.1 \times$ (age of Universe) is a time-constant characterizing the decay of the activity. 'Volume' evolution (Lynden-Bell 1971; Lynds & Wills 1972) gives very similar behaviour to 'exponential' evolution and was proposed from the observed distribution of V against V_{max}; it has no physical pretensions, and indicates that the local space density of quasars is zero. Fanti *et al.* (1975)

object; Wills & Lynds (1978) emphasize that there is no evidence to the contrary. The performances of these three functions are compared in figure 3. V/V_{max} and source–count analyses (Schmidt 1972*a*; Wills & Lynds 1978; Wall *et al.* 1977) now agree that 'power-law' evolution gives a much poorer representation of the data than do the other functions.

2.3. *Differential evolution*

So far I have confined discussion to results for radio sources found in low-frequency surveys (3CR, 4C) – the powerful radio galaxies and quasars of steep radio spectra and extended structure – and indeed the history of source–count and V/V_{max} analyses is based on such objects. Table 1 strongly suggests that other populations, and in particular weaker radio galaxies and 'flat-spectrum' quasars, follow less dramatic evolutionary patterns, and I now consider this differential evolution.

The matter of differential evolution is a triumph for source–counts. There can be little doubt that the V/V_{max} analysis is the most powerful technique for exploring radial distributions and luminosity functions of single classes of intrinsically powerful objects. Source–count analyses have a fundamental advantage in that the 'source–count argument' indicates evolution regardless of the nature of the red shifts. But in addition, each source count comprises data on all classes of the radio-source population, not just on the members that are very luminous in two wavebands. This mixture has been considered a vice by many authors; in fact it is a virtue, and the fundamental way in which source counts complement V/V_{max} analyses. For source count analyses, *complete* samples in the V/V_{max} sense are not required; as long as we have enough identifications for a class (which may be defined by radio or optical properties) to be confident about representative luminosities, then *some* information on its spatial distribution can be derived from the counts.

Since Longair's (1966) analysis, differential evolution has been recognized as an essential part of the source-count story. Longair showed that the strong evolution must be confined to the most powerful radio sources, and the reason for this is apparent in figure 2. We know that the observed range in power for radio sources is greater than 10^5, while we see bends in the count over ranges of less than 10^2 in apparent intensity. Therefore the region of the luminosity function that is allowed to evolve to produce these bends must be narrower than 10^2. The evolution function must depend on power as well as epoch; $F = F(P, z)$, so that there is a transition between low-power radio galaxies which show little or no evolution and the high-power radio galaxies and quasars whose strong evolution is satisfactorily described by the *ad hoc* functions of §2.2. If the transition power is independent of epoch, the evolution is described (regrettably) as 'luminosity-dependent density evolution'. However, $F(P, z)$ allows for an unbiased mixture of luminosity and/or density evolution, and the 408 MHz source–counts together with identification/red shift data are ideally suited to studying the transition region of this function. A successful model found in a recent and continuing investigation (Wall *et al.* 1977) has the transition power as a function of epoch, the range of powers allowed to evolve progressively narrowing to earlier epochs or larger red shifts. It therefore combines both luminosity and density evolution, similar to the proposal by Rowan-Robinson (1970*b*). On the other hand, Wall and co-workers found satisfactory models in which the transition region was *not* a function of epoch, and these models were derived by considering V/V_{max} data for the brightest sources only. Such a result emphasizes the complementary nature of V/V_{max} and source–count data.

Radio sources with steep spectra and extended structures, then, show a vast difference in their radial distribution: those of luminosities greater than $P_{408} \approx 10^{25}$ W Hz^{-1} sr^{-1} were far more plentiful at earlier epochs than now, while those of lower luminosity are approximately uniformly distributed. It is of great interest that the radio morphology of steep-spectrum sources is known to change at about this luminosity (Fanaroff & Riley 1974). The more powerful sources have their regions of highest surface brightness at the ends of the double-lobe structure, while the lower-power objects show a bewildering variety of forms in which the high surface

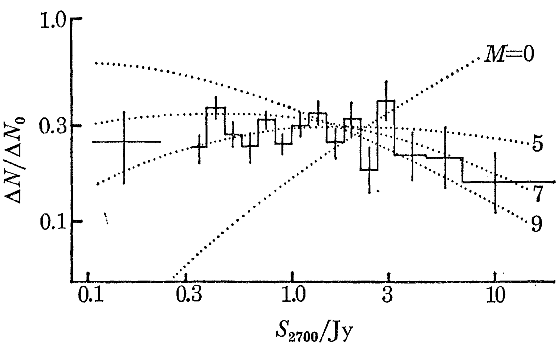

FIGURE 4. A subcount, from the Parkes 2700 MHz survey, for 'flat-spectrum' radio sources. These are of compact radio structure and the majority are identified as quasars. The dotted lines are counts computed for simple density evolution of 'exponential' type; $M = 0$ corresponds to a uniform distribution, while $M \geqslant 10$ is needed to explain source-counts and V/V_{max} data for 'steep-spectrum' quasars and powerful radio galaxies. The diagram suggests moderate evolution, $M \approx 6$, for the 'flat-spectrum', compact, radio-quasars, in agreement with results from V/V_{max} analyses.

brightnesses generally do not occur at the extremities. But the radio sources that come to dominate surveys (and counts) made at higher frequencies (figure 1) show a much more dramatic difference in structure yet: these compact sources have structures on milliarcsec scales and integrated spectra described as 'inverted', 'cm-excess', or worst of all 'flat' (never true; this is a way of saying 'not-steep'). The great majority of such sources have quasars as optical counterparts; the luminosities are therefore high and many of the objects vary in intensity at both radio and optical wavelengths on time scales as short as weeks. These levels of luminosity and activity suggest that evolution might be more extreme than for the relatively staid steep-spectrum objects.

The opposite is apparent. $\langle V/V_{max}\rangle$ results for independent samples of 'flat-spectrum' quasars show values significantly closer to 0.5 than do 'steep-spectrum' samples (table 1). These results are qualitatively consistent with the change in form of source–count with increasing frequency (figure 1) which is due to the inclusion of an increasing proportion of 'flat-spectrum' sources. That it is quantitatively consistent can be demonstrated as follows. Radio spectra obtained for sources selected in the Parkes 2700 MHz survey enable a 2700 MHz 'sub-count' to be constructed for the 'flat-spectrum' sources as shown in figure 4. The count is strikingly Euclidean; but most of its constituents are identified with quasars with large red shifts, the average luminosity is high, and the geometrical effects are strong. The count computed for a uniform distribution indeed bears no resemblance to the sub-count at all. However, equally bad fits are provided by counts for 'exponential' models in which the exponent $M \geqslant 10$, as demanded for the powerful 'steep-spectrum' quasars and radio galaxies. Moderate evolution, $M \approx 6$, is indicated, and the corresponding values for $\langle V/V_{max}\rangle$ are *ca.* 0.6, close to those obtained for 'flat-spectrum' quasars as shown in table 1. There is more to this curious story, which is resumed in §3.

A final point with regard to differential evolution: it is of interest to note that the slope that Green & Schmidt (1978) find for the $N(m)$ relation of optically selected quasars (0.93, $15.7 < B < 18.5$) is higher than the slopes found by Braccesi & Formiggini (1969) and Sandage & Luyten (1969) (0.72 and 0.75 respectively for the magnitude range *ca.* 17.5–19). If the $N(m)$ slope does change over *ca.* 3 magnitudes, then differential evolution is implied as for the radio source-counts: the optical luminosity function (see, for example, Sramek & Weedman 1978) is broader than the region of slope change, and the strong evolution must be confined to the more (optically) luminous quasars.

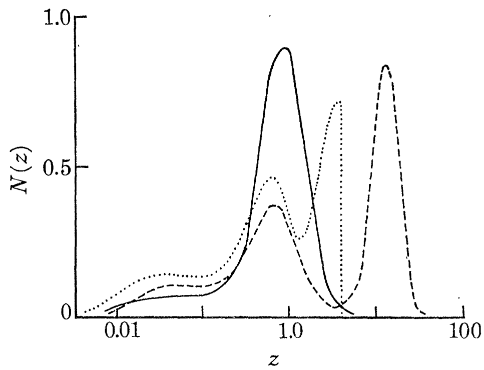

FIGURE 5. The distribution of red shifts for sources with $S_{408} = 10$ mJy, as predicted by three successful evolution models (Wall *et al.* 1977): (i) solid curve, 'exponential' evolution, with transition power dependent on epoch; (ii) broken curve, 'exponential' evolution with exponent dependent on radio power; (iii) dotted curve, as for (ii) but with a cut-off imposed at $z = 3.5$. The large variation in $N(z)$ with model shows how identification data for faint samples of radio sources will enable constraints to be placed on the models, and how such data may yield information about the reality of a red shift cut-off.

2.4. *Dark Ages*

Do our explorations of the evolution function F suggest a red shift cutoff, a cutoff corresponding to the epoch of creation of radio sources? This in turn bears on epoch of galaxy formation, commencement of nuclear activity signalling the epoch at which there were aggregates capable of forming adequate gravitational potential wells *and* of providing fuel for them.

The answer is no; all we can infer from the present V/V_{max} and source–count data is that the density cannot increase appreciably beyond $z = 2.5$. There is a conspiracy of Nature to turn earlier epochs into Dark Ages for cosmic historians of active galaxies: K-dimming plus the behaviour of volumes per Δz shell plus luminosity functions that fall steeply with luminosity. The net result is that the data are insensitive to whatever form of density behaviour we care to inflict at large red shifts. The same factors militate against demonstrating a red shift cutoff with samples of quasars found in the present prism/grism surveys, and in addition, a bias against objects with large z has been introduced with a disadvantageous grating blaze (Carswell & Smith 1978).

These previous failures have given rise to programmes which may succeed. New optical searches for quasars with large z are in progress with improved prisms/grisms (Smith 1978). For 'steep-spectrum' radio sources, analysis of current 408 MHz source–count data (Wall *et al.* 1977) show that successful models of evolution predict very different red shift distributions at faint flux levels, as indicated in figure 5. In particular, evolution models which incorporate red shift cutoffs predict relatively large proportions of the faint radio sources to have red shifts less than 0.5; if such models are right, the identification content of faint samples will be high. To this end, a new 5C survey (5C12; S_0 (408 MHz) = 10 mJy; $(\alpha, \delta)_{1950}$ = 13 h, +35°) by C. R. Benn (M.R.A.O.) is complete, and in collaboration with G. Grueff and M. Vigotti (Laboratorio di Radioastronomia, C.N.R., Bologna), deep plates taken on the Palomar Schmidt telescope are being searched at the radio positions. Finally, the 'flat-spectrum' radio sources hold out the most promise, because the radio K-dimming is less while the radio luminosity function is flatter than for the 'steep-spectrum' sources (Masson & Wall 1977; Wills & Lynds 1978). Indeed, the dramatic effect of inflicting a red shift cutoff in simple density-evolution models is shown in figure 7 of Wall (1978). The most pressing observational requirement for extracting 'Dark Ages' information from high-frequency source counts is the definition of a complete sample of bright sources selected at a high frequency, and the construction of a luminosity distribution from these sources and their optical data. In this regard, two complete samples have been defined at 2700 MHz (Peacock & Wall 1979), an all-sky catalogue comprising 229 sources (S_0 (2700 MHz) = 2.0 Jy), and a northern sample comprising 161 sources (S_0 (2700 MHz) = 1.5 Jy). In each of these the identification rate is *ca.* 85 %, high enough to construct luminosity distributions to explore the implications of the 2700 MHz counts in the manner proposed by Wall *et al.* (1977).

The present (and extremely weak) indications are that the 'steep-spectrum' radio sources may display a red shift cutoff, while the 'flat-spectrum' sources (i.e. the radio-compact quasars) may not. It is far too early to speculate about which came first, about whether the one is the progenitor of the other, or even whether we shall eventually succeed in establishing the epoch of creation of any active-galaxy population.

3. Physical considerations

There can be no satisfactory physical description of the evolution until we have satisfactory physical models of quasars and radio galaxies, but it is also clear that for these physical models to be entirely satisfactory, they must result in (or at least allow) the observed radial distributions, complete with evolution and differential evolution. The evolution has suggested a number of physical considerations, which take one of two forms. The first of these I term 'semi-physical': the differing evolution properties of the active-galaxy subpopulations are used to propose a general evolutionary scheme, some objects as progenitors, others as products, accounting for observed radial distributions with many fewer parameters than required by descriptions of the individual distributions of the different populations. The second form of consideration is 'real-physical': some (partly understood) astrophysics is used to explain (or is shown *not* to explain) some aspect of the radial distribution or evolution.

3.1. *Semi-physical models*

With regard to semi-physical models, Grueff & Vigotti (1977) propose an interesting scheme in which all radio sources are born as quasars at z = 2.5, with a quasar essentially a super-active nucleus of a giant elliptical galaxy. The comoving density of quasars decreases exponentially with epoch, and when a quasar dies, it dies optically, leaving a radio galaxy that lives for a time proportional to $1/P_{\text{rad}}$. A model based on these assumptions, a guessed radio luminosity function for quasars at $z = 2.5$, and four parameters, yields remarkably good agreement with low-frequency source counts and the identification data, particularly the proportions of quasars and radio galaxies detected at different flux-density levels. Differential evolution between the powerful and the weak radio galaxies comes about because the density of radio galaxies decreases with a time-constant depending on the radio power; radio galaxies of low luminosity have time-constants longer than t_0, resulting in little or no change of density with epoch. These long lifetimes may be a stumbling-block for the model; additionally, the model does not incorporate either the radio-quiet quasars, or the 'flat-spectrum' radio-quasars which so dominate the statistics of high-frequency radio surveys. A much more ambitious scheme is proposed by Rowan-Robinson (1977), who attempts to unite the luminosity functions of quasars and radio galaxies with quiescent galaxies via the use of probability functions; the scenario in its present form requires some 30 parameters, and does not yet incorporate evolution at all.

3.2. *Hints from the evolution functions*

With regard to physics suggested by the evolution functions that have been proposed (§2.2), there is an inverse correlation between the fit of the function to the data and the amount of physical interpretation to which it has been subjected. 'Volume' evolution provides the best representation of the data; physical significance is least obvious. 'Exponential' evolution is also very satisfactory, and there are several references to the time-scale for decay of activity which it suggests (*ca.* 0.1 t_0 for powerful radio sources with extended structures). Schmidt (1972*b*) goes further: he showed that (i) if giant ellipticals began forming at $t = 0$ at a rate $C \exp[-M(t/t_0)]$ and (ii) if each is a radio galaxy for T years after formation, then the density of radio galaxies increased with look-back-time up to $t_0 - T$, i.e. $F = \exp[M/t_0(t_0-t)]$ for $z < z(T)$. The 'exponential' law therefore arises naturally, the exponent being that of the decay of formation rate for giant ellipticals; but given the need for the initial assumptions, it is not obvious that much has been gained.

'Power-law' evolution, $F = (1+z)^n$, provides the worst representation of $V/V_{\max}$ and source-count data, and has yielded by far the largest amount of physical discussion. It is clear why this should be so, since matter density, energy density, and temperature all scale as some power of the universal scale factor $R = (1+z)^{-1}$. There are two obvious ways in which the i.g.m. might produce evolutionary effects for the extended radio sources: Compton snuffing and ram-pressure confinement.

(*a*) *Compton snuffing*. The fate of extended radio sources in the hands of the microwave background has been considered by several authors (e.g. Rees & Setti 1968; Rowan-Robinson 1970*b*; Scheuer 1977). In the lobes, inverse Compton losses must be suffered by the relativistic electrons as they impinge on the background photons. The energy density of this background is proportional to $(1+z)^4$, and differential evolution would arise because the weaker, diffuse sources would have correspondingly shorter lifetimes at the earlier epochs. It is difficult to produce enough differential evolution in this way (Scheuer 1977), and in any case the density enhancement at early epochs must be postulated *ab initio*. But with regard to this enhancement, Rees & Setti (1968) point out that a red shift cutoff could arise at $z \gtrsim 4$ by complete Compton snuffing. There may therefore be no need for the formation–evolution function to have a built-in 'epoch of creation', and it is imporantt to note that such a Compton cutoff applies to the extended 'steep-spectrum' sources alone (see §2.4).

(*b*) *Ram-pressure confinement*. The density of the i.g.m. is proportional to $(1+z)^3$; if ram-pressure is of critical importance to extended radio sources, then the earlier, denser epochs of the Universe might greatly favour higher formation rates and/or longer lifetimes. However, no steep dependence on $(1+z)$ has been demonstrated; Rees & Setti (1968) obtained a density dependence for spherical structures of only $(1+z)^{1.5}$. That the i.g.m. density does play a role is suggested by the relatively weak dependence of density of *compact* radio sources on epoch. However, there are three indications to the contrary. First, if the i.g.m. were important, most or all extended radio sources should be in clusters, where there is good evidence that the ambient medium is of density higher than average. In fact luminosity functions of cluster and non-cluster radio galaxies do not differ significantly (Jaffe & Perola 1976), although it should be noted that comparisons have been limited to the lower radio powers. Secondly, the strong evolution inferred for optically selected quasars (which are radio-quiet for the most part) clearly has nothing to do with the i.g.m. Finally, the dependence of radio luminosity function on the optical luminosity (Schmidt 1970) *for extended radio quasars* indicates that it is the activity at the nucleus that governs the existence of the sources; the influence of the i.g.m. on birth rate or lifetime must be secondary. It seems much more likely that epoch-dependence of fuel supply or of *some* property of the central powerhouse provides most of the evolutionary effects.

In this regard, and still with $F = (1+z)^n$ in mind, the work of Gisler (1976) is of particular interest. He considered the fate of gas shed by stars evolving in elliptical galaxies, and showed that (i) the rate of production is proportional to t^{-1}, where t is the age of the galaxy, and (ii) the accumulation must exceed present observational limits unless there is a removal mechanism. If violent activity in nuclei is fuelled by continuous gas supply, then the decline of gas production at rate t^{-1} is much too gentle to produce the observed density evolution of powerful radio sources. But if the necessary removal mechanism is more efficient at later times, then the correspondingly steep drop in fuelling rate could produce the evolution. Sweeping the gas from ellipticals by ram-pressure interaction with the i.g.m. is just such a mechanism; Gisler found that the density of galaxies that have accumulated enough gas to drive a nuclear engine is proportional to $t^{-5.25}$ for low-velocity galaxies and to $t^{-2.33}$ for high-velocity galaxies. In a Friedmann Universe, such a density dependence for the low-velocity galaxies translates to $F = (1+z)^{7.9}$ if $\Omega = 1$, or to $F = (1+z)^{5.3}$ if $\Omega = 0$. The emergence of 'power law' evolution from the scheme is an artefact due to choice of $\Omega = 0$ or $\Omega = 1$ to translate epoch into red shift, and Gisler pointed out that in any case there are moderating factors that might change the form of the law.

A particularly attractive feature of the scheme is that the low-velocity (massive) galaxies, with which the *powerful* radio sources are associated, show a much steeper density law then the high-velocity galaxies, just as required for the differential evolution inferred from source–counts. The scenario has one serious difficulty as Gisler notes: the sweeping ought to be very much more effective in clusters. It is not clear that this can be reconciled with the similarity between radio-luminosity functions for cluster and non-cluster sources, although there are ways out of the difficulty.

3.3. *The Scheuer–Readhead model for compact radio sources*

Lastly I consider the cosmological implications of the model put forward by Scheuer & Readhead (1979) for superluminally expanding radio sources. Their model for compact sources has radio-emitting blobs ejected quasi-continuously along one axis at speeds $v \sim c$; the relativistic speeds result in apparent transverse velocities greater than c, and in highly asymmetric structures. Both features are seen in most of the (few) sources mapped with very long baseline interferometric (v.l.b.i.) techniques. The most appealing feature of the model is that to be observed as such a radio source, the ejection axis must (nearly) coincide with our line of sight. If it is misaligned, the object is a radio-quiet quasar to us, and distinction between radio-quiet and radio-loud quasars therefore becomes the simple geometric matter of aiming the radio axis properly. The surface densities of radio-quiet and radio-loud quasars are known to be in the ratio 10–100:1, a ratio that Scheuer & Readhead showed to be in accord with the prediction of their model.

There are three cosmological aspects, the first two of which have been aired by Scheuer & Readhead.

(*a*) *A broadened luminosity function*. The relativistic projection effect produces an enormous broadening of the 'true' radio luminosity function, the ratio of P_{obs}/P_{real} for the approaching blobs being $[\gamma(1-v_{app}/c)]^{-(2+\alpha)}$, where $\gamma = (1-(v/c)^2)^{-\frac{1}{2}}$, v_{app} is the component of ejection velocity along the line of sight, and α is the radio spectral index. Such a broadened luminosity function might flatten the 'evolutionary hump' observed in low-frequency source counts. However, a simple convolution shows that the effect is not significant; certainly it cannot produce the relatively flat subcount of figure 4 for compact sources from a low-frequency count of the form shown in figure 2.

(*b*) *Intrinsically low luminosities*. The compact quasars have high *observed* luminosities, boosted from their intrinsic powers by the radio P_{obs}/P_{real} given above. This factor can approach 10^5 for $\gamma = 5$ and therefore P_{real} for these objects drops them down to luminosities typical of 'weak' radio galaxies. Such sources show relatively little change in space density with epoch (see §2.3); the low values of $\langle V/V_{max}\rangle$ and the form of the source count for the compact quasars suggest that these objects (turned into low-power sources on the Scheuer–Readhead model) obey a similar radial distribution.

(*c*) *Two classes of compact radio quasar?* The model thus provides a link between the relatively mild evolution observed for weak radio galaxies and for compact radio quasars. Such a link would be comforting *if* the radio-quiet quasars did not appear to evolve strongly. Why do the compact quasars not mimic *this* evolutionary behaviour if they are the same objects except for a geometrical accident?

One way out is the following. Among the compact quasars, two types can be distinguished on the basis of their integrated spectra. The first of these (prototype 3C 273; fig. 7 of Wall 1972) has considerable flux density at the lower frequencies, suggesting some extended structure, together with enhanced and highly variable emission at high frequencies, indicating rapidly evolving components of high surface brightness. The second type (prototype PKS 2134–004; fig. 11 of Wall 1972) has a simpler spectrum, a single hump with no low-frequency components, and shows slow, regular variations of only a small percentage at the high frequencies. On the basis of very small samples, Masson & Wall (1977) pointed out that $\langle V/V_{max}\rangle$ and source subcounts suggested evolution for 3C 273-type objects and an approximately uniform distribution for the PKS 2134–004-type objects. Of crucial importance are v.l.b.i. observations to determine if members of the PKS 2134–004 class show superluminal expansions and asymmetric structures. If they do not, then it may be that the 3C 273-type quasar, despite intrinsically low power on the Scheuer–Readhead model, follows the strong evolution of powerful radio galaxies, 'steep-spectrum' quasars, and radio-quiet quasars, while the PKS 2134 − 004-type quasar obeys a near-uniform radial distribution, high luminosities notwithstanding.

The differences in evolution noted by Masson & Wall may disappear with larger samples. It is then possible (and attractive) that a single luminosity function plus dispersion in γ can describe all quasars via the Scheuer–Readhead model. However, if structural and cosmological differences persist, we may have to consider that central engines of the two types of compact radio quasar differ in some fundamental way.

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Discussion

D. MEIER (*Institute of Astronomy, Cambridge, U.K.*). In addition to the V/V_{max} and source–count methods for studying the evolution of radio sources there is the luminosity function method in which one identifies large numbers or radio sources down to given optical and radio limits and computes a radio luminosity function (r.l.f.) in different red shift ranges. Eventually this method will give the form of the evolution without the assumptions that one needs to make in the source count analysis. However, it involves a large amount of radio and optical work and, as a result, it will be saome time before very deep samples to large red shifts are obtained. Recently (1979, *Astrophys. J.* **229**, 25), M.-H. Ulrich of E.S.O. and R. Fanti, I. Gioia, and C. Lari of the Bologna group and I have analysed three complete samples of radio galaxies that have a significant number of objects around $z = 0.1$ and we find no evolution in the r.l.f. out to this red shift. This essentially confirms the source–count analysis which *predicts* slow evolution for these red shifts. However, deeper samples would be able to detect significant evolution if the source–count analysis is also correct for larger red shifts.

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